Flow Equation Approach to a Singular SPDE

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Motivation

Many SPDEs coming from physics are singular, we cannot use classical tools.

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EXAMPLE : Dynamical Φ^4_d model

$$(\partial_t + 1 - \Delta)\Phi = \xi - \lambda \Phi^3$$

posed in $\mathbb{R}_+ \times \mathbb{R}^d$ for $d \in \{2,3\}$

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EXAMPLE : Dynamical Φ^4_d model

$$(\partial_t + 1 - \Delta)\Phi = \xi - \lambda \Phi^3$$

$\leftarrow$ when $\lambda = 0$ we expect Φ to be a distribution, so Φ^3 is not well defined

posed in $\mathbb{R}_+ \times \mathbb{R}^d$ for $d \in \{2, 3\}$

Motivation

We focus on a simpler equation

$$(1-\Delta)^{\sigma/2}\Phi = \xi + \lambda \Phi^3$$

How can we give meaning to it?

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$$(1-\Delta)^{\sigma/2}\Phi = \xi + \lambda\Phi^3 \qquad (1)$$

How can we give meaning to it?

First we regularize the noise: $\theta \in C_c^\infty(\mathbb{R}^d)$

$$\theta_K(x) = [K]^d\,\theta\left(\frac{x}{[K]}\right) \qquad [K] = K^{\frac{1}{\sigma}}$$

$$\rightsquigarrow \quad \xi_K = \theta_K * \xi \in C^\infty(\mathbb{R}^d)$$

Motivation

How can we give meaning to it?

Use it to define

$$\Phi_K = G * F_K[\Phi_K] \quad \text{where}$$

- G is the green function of $(1-\Delta)^{\sigma/2}$
- $F_K[\varphi](x) = \xi_K(x) + \lambda\varphi(x)^3$

Motivation

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$$\Phi_\kappa = G * F_\kappa[\Phi_\kappa] \quad \text{where} \qquad (2)$$

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- $F_\kappa[\varphi](x) = \xi_\kappa(x) + \lambda\varphi(x)^3$

Now we hope on some convergence of Φ_κ as $\kappa \downarrow 0$

DOES NOT WORK !

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- G is the green function of $(1-\Delta)^{\sigma/2}$
- $F_\kappa[\varphi](x) = \xi_\kappa(x) + \lambda\varphi(x)^3 + \sum_{i=1}^{i^*} \lambda^i c_\kappa^{(i)} \varphi(x)$

Now we hope on some convergence of Φ_κ as $\kappa \downarrow 0$

NOTATION $$(1-\Delta)^{\sigma/2}\Phi = \xi + \lambda\Phi^3 - \infty\,\Phi$$

Main theorem

THM let $d \in \{1 \dots 6\}$ and $\sigma \in \left(\frac{d}{3}, \frac{d}{2}\right]$, then there exists a choice of the counterterms, a random variable $\lambda_* \in [0,1]$ and $\Phi \in \mathcal{S}'(\mathbb{R}^d)$ such that:

- for every random variable $\lambda \in [-\lambda_*, \lambda_*]$ there exists Φ_K that solves (2)
- $\Phi_K \longrightarrow \Phi$ almost surely in the $\mathcal{S}'(\mathbb{R}^d)$ topology
- $\mathbb{E}[\lambda_*^{-m}] < +\infty$ for all $m \in \mathbb{N}_+$

We will say that Φ is a solution of (1)

The role of σ

$$\mathcal{C}^{\alpha}(\mathbb{T}^d) \equiv B^{\alpha}_{\infty,\infty}(\mathbb{T}^d)$$

$\alpha > 0$: regular Hölder space

$\alpha \leq 0$: $\phi \in \mathcal{S}'(\mathbb{T}^d)$ st $\|\phi\|_{\mathcal{C}^{\alpha}} < +\infty$

with $\|\phi\|_{\mathcal{C}^{\alpha}} = \sup_{\mu \in (0,1)} [\mu]^{-\alpha} \|K_\mu * \phi\|$

$$K_\mu(x) = [\mu]^{-d} K\left(\frac{x}{[\mu]}\right)$$

We say that ϕ has regularity α if $\phi \in \mathcal{C}^{\alpha}(\mathbb{T}^d)$

The role of σ

PROPERTIES OF $\mathcal{C}^\alpha$

- there is a continuous immersion $\mathcal{C}^\alpha \hookrightarrow \mathcal{C}^\beta$ for every $\alpha \geq \beta$

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PROPERTIES OF $\mathcal{C}^\alpha$

- there is a continuous immersion $\mathcal{C}^\alpha \hookrightarrow \mathcal{C}^\beta$ for every $\alpha \geq \beta$
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- ξ is in $\mathcal{C}^{-d/2}$ (almost)

The role of σ

PROPERTIES OF $\mathcal{C}^\alpha$

- there is a continuous immersion $\mathcal{C}^\alpha \hookrightarrow \mathcal{C}^\beta$ for every $\alpha \geq \beta$
- the convolution with G maps $\mathcal{C}^\alpha \to \mathcal{C}^{\alpha+\sigma}$
- ξ is in $\mathcal{C}^{-d/2}$ (almost)
- if $\alpha+\beta>0$ then the product of two functions extends to $\mathcal{C}^\alpha \times \mathcal{C}^\beta \to \mathcal{C}^{\alpha\wedge\beta}$

The role of σ

If Φ solves $\Phi = G * \xi$ then Φ has regularity

$$\alpha = \sigma - \frac{d}{2}.$$

Assuming that Φ^3 is a small perturbation then also Φ solving (1) should have regularity α

For $\alpha > 0$, i.e. $\sigma > \frac{d}{2}$, there are no problems.

When $\sigma \leq \frac{d}{3}$ the assumption above does not hold anymore.

In general, small σ implies less regularity.

Da Prato-Debussche regime

We try to solve $\Phi_\kappa = G * (\xi_\kappa + \lambda \Phi^3 + \lambda c_\kappa^{(1)} \Phi_\kappa)$

Make the ansatz $\Phi_\kappa = ?_\kappa + \Psi_\kappa$, $\quad ?_\kappa = G * \xi_\kappa$

Da Prato-Debussche regime

We try to solve $\Phi_K = G * (\xi_K + \lambda \Phi^3 + \lambda c_K^{(1)} \Phi_K)$

Make the ansatz $\Phi_K = \mathsf{I}_K + \Psi_K$, $\quad \mathsf{I}_K = G * \xi_K$

Then $\Psi_K = \lambda G * (\Psi_K^3 + 3\Psi_K^2 \mathsf{I}_K + 3\Psi_K \mathsf{V}_K + \mathsf{W}_K)$

$\mathsf{V}_K = (\mathsf{I}_K)^2 - c_K^{(1)}/3 \qquad \mathsf{W}_K = (\mathsf{I}_K)^3 + c_K^{(1)} \mathsf{I}_K$

$c_K^{(1)} = -3\mathbb{E}(\mathsf{I}_K)^2$

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$(\mathrm{I}_K, \mathrm{V}_K, \Psi\!\!\!\vert_K) \xrightarrow{K \to 0} (\mathrm{I}, \mathrm{V}, \Psi\!\!\!\vert) \in \mathcal{C}^{\alpha} \times \mathcal{C}^{2\alpha} \times \mathcal{C}^{3\alpha}$

We can find a fixed point of the map

$$Q[\varphi] = \lambda G * (\varphi^3 + 3\varphi^2 \mathrm{I} + 3\varphi \mathrm{V} + \Psi\!\!\!\vert)$$

We need $3\alpha + \sigma > 0 \rightsquigarrow \sigma > \frac{3d}{8}$

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$$Q[\varphi] = \lambda G * (\varphi^3 + 3\varphi^2 \,\mathsf{I} + 3\varphi \vee + \Psi)$$

$$Q . \ \mathcal{C}^{3\alpha+\sigma} \longrightarrow \mathcal{C}^{3\alpha+\sigma}$$

$\varphi^2 \,\mathsf{I}$, $\varphi \vee$ are well defined if

$(3\alpha + \sigma) + \ell\alpha > 0$
for $\ell \in \{1, 2\}$ $\Longrightarrow$ $\boxed{\sigma > \frac{5d}{12}}$

Da Prato-Debussche regime

We try to solve $\Phi_K = G * (\xi_K + \lambda \Phi^3 + \lambda c_K^{(1)} \Phi_K)$

We have a solution for the equation for

$$\sigma \in \left(\frac{5d}{12}, \frac{d}{2}\right].$$

Repeating the trick we can arrive to $\sigma \in \left(\frac{2d}{5}, \frac{d}{2}\right]$.

This is called the Da Prato-Debussche trick.

we need still to add more counterterms

Effective equation

We go back to $\Phi_k = G * F_k[\Phi_k]$, to study it we make a "scale decomposition"

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Fix $[0,1] \ni \mu \mapsto G_\mu \in L^1$ st $G_0 = G$ and $G_1 = 0$, use it to define

$$\Phi_{k,\mu} = G_\mu * F_k[\Phi_k] \qquad \text{coarse grained process}$$

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We go back to $\Phi_\kappa = G * F_\kappa[\Phi_\kappa]$, to study it we make a "scale decomposition"

Fix $[0,1] \ni \mu \mapsto G_\mu \in L^1$ st $G_0 = G$ and $G_1 = 0$, use it to define

$$\Phi_{\kappa,\mu} = G_\mu * F_\kappa[\Phi_\kappa]$$ coarse grained process

Fix also $F_{\kappa,\mu}$ st $F_{\kappa,0} = F_\kappa$ and define

$$\mathcal{E}_{\kappa,\mu} = F_\kappa[\Phi_\kappa] - F_{\kappa,\mu}[\Phi_{\kappa,\mu}]$$

Effective equation

Deriving and re-sobstituting we get the system:

$$\begin{cases} \Phi_{k,\mu} = -\int_\mu^1 \dot{G}_\eta * \left(F_{k,\eta}[\Phi_{k,\eta}] + \Xi_{k,\eta}\right) d\eta \\ \Xi_{k,\mu} = -\int_0^\mu \left(H_{k,\eta}[\Phi_{k,\eta}] + DF_{k,\eta}[\Phi_{k,\eta}]\cdot\left(\dot{G}_\eta * \Xi_{k,\eta}\right)\right) d\eta \end{cases}$$

with $\quad H_{k,\mu}[\varphi] = \partial_\mu F_{k,\mu}[\varphi] + DF_{k,\mu}[\varphi]\cdot\left(\dot{G}_\mu * F_{k,\mu}[\varphi]\right)$

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with $\quad H_{k,\mu}[\varphi] = \partial_\mu F_{k,\mu}[\varphi] + DF_{k,\mu}[\varphi] \cdot \left(\dot{G}_\mu * F_{k,\mu}[\varphi]\right)$

With the appropriate choice of G_μ, $F_{k,\mu}$ we can keep this meaningful also for $k \to 0$

Effective equation

Using an appropriate kernel K_μ we can rewrite

$$\begin{cases} \tilde{\Phi}_{\kappa,\mu} = -\int_\mu^1 K_{\mu,\eta} * \dot{\tilde{C}}_\eta * \left(\tilde{F}_{\kappa,\eta}[\tilde{\Phi}_{\kappa,\eta}] + \tilde{\Xi}_{\kappa,\eta}\right) d\eta \\ \tilde{\Xi}_{\kappa,\mu} = -\int_0^\mu K_{\mu,\eta} * \left(\tilde{H}_{\kappa,\eta}[\hat{\Phi}_{\kappa,\eta}] + D\tilde{F}_{\kappa,\eta}[\hat{\Phi}_{\kappa,\eta}] \cdot (\tilde{C}_\eta * \Xi_{\kappa,\eta})\right) d\eta \end{cases} \qquad (3)$$

$\tilde{F}_{\kappa,\mu}[\varphi] := K_\mu * \hat{F}_{\kappa,\mu}[\varphi]$ $\quad \Phi_{\kappa,\mu} = K_\mu * \hat{\Phi}_{\kappa,\mu}$ $\quad \tilde{\Xi}_{\kappa,\mu} = K_\mu * \Xi_{\kappa,\mu}$

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$$\|\tilde{\Phi}_{k,\mu}\| \lesssim \int_\mu^1 \|K_\eta * F_{k,\eta}[\tilde{\Phi}_{k,\eta}]\| d\eta$$

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$$\| \tilde{\Phi}_{k,\mu} \| \lesssim \int_\mu^1 \| K_\eta * F_{k,\eta} [\tilde{\Phi}_{k,\eta}] \| d\eta$$

$$\sim \int_\mu^1 \| K_\eta * \Xi_k \| d\eta$$

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Independent of κ!

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Using an appropriate Kernel K_μ we can rewrite

$$\tilde{\mathcal{E}}_{\kappa,\mu} = -\int_0^\mu K_{\mu,\eta} * \left(\tilde{H}_{\kappa,\eta}[\hat{\Phi}_{\kappa,\eta}] + D\tilde{F}_{\kappa,\eta}[\hat{\Phi}_{\kappa,\eta}] \cdot (\tilde{C}_\eta * \mathcal{E}_{\kappa,\eta})\right) d\eta$$

$$\| \tilde{\mathcal{E}}_{\kappa,\mu} \| \lesssim \int_0^\mu \| \tilde{H}_{\kappa,\eta}[\tilde{\Phi}_{\kappa,\eta}] \| d\eta \lesssim \int_0^\mu [\eta]^{\beta-\sigma} \lesssim [\mu]^\beta$$

We need $\beta > 0$ to do this !

Indeed it holds that:

Effective equation

THM the system (3) has a solution for every $\kappa>0$ if we have some control on $\tilde{F}$, $\tilde{H}$. In particular we want that:

$$[\mu]^{\alpha-\sigma} \|\tilde{F}_{\kappa\mu}[\varphi]\| \leq R\left(\frac{1}{2} + |\lambda|^{1/3}[\mu]^{-\alpha}\|\varphi\|\right)^{m_b}$$

$$[\mu]^{\beta-\sigma} \|\tilde{H}_{\kappa\mu}[\varphi]\| \leq R^2 |\lambda|^{1/3}\left(\frac{1}{2} + |\lambda|^{1/3}[\mu]^{-\alpha}\|\varphi\|\right)^{m_b}$$

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So now: construct $F_{\kappa,\mu}$, $H_{\kappa,\mu}$ st $\tilde{F}_{\kappa,\mu}$, $\tilde{H}_{\kappa,\mu}$ satysfy the condition ! The hardest part will be having $\beta > 0$.

Effective force coefficients

We start with the ansatz

$$\langle F_{\kappa,\mu}[\varphi], \psi \rangle = \sum_{i=1}^{i_b} \sum_{m=1}^{3i} \lambda^i \langle F_{\kappa,\mu}^{i,m}, \psi \otimes \varphi^{\otimes m} \rangle$$

for some $F_{\kappa,\mu}^{i,m} \in \mathcal{S}'(\mathbb{H}^{1+m})$ (here $\mathbb{H} = \mathbb{R}^d$)

Effective force coefficients

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for some $F_{\kappa,p}^{i,m} \in \mathcal{S}'(\mathbb{M}^{1+m})$ (here $\mathbb{M} = \mathbb{R}^d$)

For example, since $F_\kappa[\varphi] = \xi_\kappa(x) + \lambda \varphi(x)^3 + \sum_{i=1}^{i_\#} \lambda^i c_\kappa^{(i)} \varphi(x)$:

$F_\kappa^{0,0}(x) = \xi_\kappa(x)$ $\quad$ $F_\kappa^{1,3}(x, dy_1 \ldots dy_3) = \delta_x(dy_1) \ldots \delta_x(dy_3)$

for $i = \{1 \ldots i^\#\}$ $\quad$ $F_\kappa^{i,1}(x, dy_1) = c_\kappa^{(i)} \delta_x(dy_1)$

Effective force coefficients

For example, since $F_k[\varphi] = \Xi_k(x) + \lambda\varphi(x)^3 + \sum_{i=1}^{i_*} \lambda^i c_k^{(i)} \varphi(x)$:

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$$\text{for } i = \{1 \ldots i^{\#}\} \qquad F_k^{i,1}(x, dy_1) = c_k^{(i)} \delta_x(dy_1)$$

Then seems natural that $F_{k,p}^{i,m} \in \mathcal{V}^m$, where

$$\mathcal{V}^m = \{V : \mathbb{H} \times \mathrm{Borel}(\mathbb{H}^m) \longrightarrow \mathbb{R}\}$$

$$\|V\|_{\mathcal{V}^m} = \sup_{x \in \mathbb{H}} \int_{\mathbb{H}^m} |V(x; dy_1 \ldots dy_m)|$$

Effective force coefficients

Remember that $H_{k,\mu}[\varphi] = \partial_\mu F_{k,\mu}[\varphi] + DF_{k,\mu}[\varphi]\cdot(\dot{G}_\mu * F_{k,\mu}[\varphi])$

$\rightsquigarrow$ we try to make this $\mathcal{O}(\lambda^{i_s+1})$, so we ask:

$$\partial_\mu F_{k,\mu}^{i,m} = -\sum_{j=0}^{i}\sum_{l=0}^{m}(1+l)B(\dot{G}_\mu, F_{k,\mu}^{j,1+l}, F_{k,\mu}^{i-j,m-k})$$

Effective force coefficients

$\rightsquigarrow$ we try to make $H_{k,\mu}[\varphi]$ an $\mathcal{O}(\lambda^{i_0+1})$, so we ask:

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Recursive construction

I) set $F_{k,\mu}^{0,0} = \xi_k$ and $F_{k,\mu}^{i,m} = 0$ for $m > 3i$

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I) set $F_{k,\mu}^{0,0} = \xi_k$ and $F_{k,\mu}^{i,m} = 0$ for $m > 3i$

II) if $F_{k,\mu}^{i,m}$ is defined for $i < i_0$ or $i = i_0$ and $m > m_0$ then

define $\dot{F}_{k,\mu}^{i_0,m_0}$ as above

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define $\dot{F}_{k,\mu}^{i_0,m_0}$ as above

III) define $F_{k,\mu}^{i_0,m_0} = F_k^{i_0,m_0} + \int_0^\mu \dot{F}_{k,\eta}^{i_0,m_0}\, d\eta$

Effective force coefficients

$c_\kappa^{(i)}$, $i \in \{1 \dots i_\#\}$

THM there exists a choice of the counterterms $\checkmark$ such that

$$\| K_\mu^{\otimes(1+m)} * F_{\kappa,\mu}^{i,m} \|_{V^m} \le \tilde{R}\,[\mu]^{\varrho(i,m)}$$

for every $i \in \{0 \dots i_b\}$ $m \in \{0 \dots 3 i_b\}$, $\kappa, \mu \in (0,1]$

where - $\varrho(i,m) = \alpha - \sigma - m\alpha + i\gamma$

- i_b is the smallest natural st $\varrho(i_b+1, 0) > 0$

- $i_\#$ is the smallest natural st $\varrho(i_\#+1, 1) > 0$

Effective force coefficients

From the last theorem the estimates that we needed follows.

This let us construct Φ_κ as in the theorem, then we would still need to study the convergence as $\kappa \to 0$, but this can be done using the properties that we have.

BIBLIOGRAPHY

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